Inverse Trigonometric Functions

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (A) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false and Reason (R) is true

Q1.

Assertion (A): All trigonometric functions have their inverses over their respective domains.
Reason (R): The inverse of $\tan^{-1} x$ exists for some $x \in R$. *(CBSE 2023)*

Answer : (d) Assertion (A) is false and Reason (R) is true

Q2.

Assertion (A): The value of $\sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is 1.
Reason (R): $\tan^{-1}(-x) = -\tan^{-1} x,\ x \in R$
and $\cos^{-1}(-x) = \cos^{-1} x,\ x \in [-1, 1]$

Answer : (c) Assertion (A) is true but Reason (R) is false

Q3.

Assertion (A): If $2(\sin^{-1} x)^2 - 5(\sin^{-1} x) + 2 = 0$, then x has 2 solutions.
Reason (R): $\sin^{-1}(\sin x) = x$, if $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

Answer : (d) Assertion (A) is false and Reason (R) is true

Q4.

Assertion (A): The domain of the function $\sec^{-1} 2x$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$.

Reason (R): $\sec^{-1}(-2) = \frac{\pi}{4}$ *(CBSE SQP 2022-23)*

Answer : (c) Assertion (A) is true but Reason (R) is false

Q5.

Assertion (A): If $\alpha \in \left(-\frac{\pi}{2}, 0\right)$, then

$$\sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha) = 0.$$

Reason (R): If $\alpha \in \left(-\frac{\pi}{2}, 0\right)$, then $\sin^{-1}(\sin \alpha) = \alpha$ and $\cos^{-1}(\cos \alpha) = -\alpha$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)

Q6.

Assertion (A): Range of $f(x) = \cot^{-1}(2x - x^2)$ is $(0, \pi)$.

Reason (R): $\cot^{-1} x$ is defined for all $x \in R$.

Answer : (d) Assertion (A) is false and Reason (R) is true

Q7.

Assertion (A): The domain for

$$f(x) = \sin^{-1}\left(\frac{1 + x^2}{2x}\right) \text{ is } \{0, 1\}.$$

Reason (R): $\sin^{-1} x$ is defined only if $x \in [-1, 1]$.

Answer : (d) Assertion (A) is false and Reason (R) is true

Q8.

Assertion (A): Principal value of $\sin^{-1}\left\{\sin\left(\frac{2\pi}{3}\right)\right\}$ is $\frac{\pi}{3}$.

Reason (R): Interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ lies in principal value branch of arc sine function.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)

Q9.

Assertion (A): Range of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$ is $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$.

Reason (R): $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$ is defined for all $x \in [-1, 1]$.

Answer : (c) Assertion (A) is true but Reason (R) is false

Q10.

Assertion (A) We can write $\sin^{-1} x = (\sin x)^{-1}$.

Reason (R) Any value in the range of principal value branch is called principal value of that inverse trigonometric function.

Q11.

Assertion (A) The inverse of sine function is define in the interval $[-\pi, 0]$, $[0, \pi]$ etc.

Reason (R) The inverse of sine function is denoted by $\sin^{-1}$.

ANSWER KEY

Q10. (d) Q11. (d)

SOLUTION

Q10. **Assertion** $\sin^{-1} x$ should not be confused with $(\sin x)^{-1}$. Infact $(\sin x)^{-1} = \frac{1}{\sin x}$ and similarly for other trigonometric functions.

Reason The value of an inverse trigonometric function which lies in the range of principal branch, is called the principal value of that inverse trigonometric function.

Hence, we can say that Assertion is false and Reason is true.

Q11. **Assertion** Sine function is one-one and onto in the interval $\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right], \left[\frac{-\pi}{2}, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ etc; and its range is $[-1, 1]$.

So, inverse of sine function is define in each of these intervals.

Reason We denote the inverse of sine function by $\sin^{-1}$ (arc sine function).

Hence, we can say that the Reason is true and Assertion is false.

$\therefore$ Both A and R are true but R is not correct explanation of A.

Assertion (A): $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Ans. Option (D) is correct.

Explanation:

The principal value branch of $\sin^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Let $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$\sin^{-1}(\sin\theta) = \sin^{-1}x = \theta$

$\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

Hence R is true.

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$, since $\frac{2\pi}{3} \notin \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Hence A is false.

Assertion (A): Range of $\tan^{-1}x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Reason (R): Domain of $\tan^{-1}x$ is R.

Ans. Option (B) is correct.

Explanation: Domain of tan x is the set $\{x : x \in R$ and $x \neq (2n+1)\frac{\pi}{2}, n \in Z\}$ and Range is R.

$\Rightarrow$ tan x is not defined for odd multiples of $\frac{\pi}{2}$.

If we restrict the domain of tangent function to $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, then it is one-one and onto with its range as R. Actually tan x restricted to any of the intervals $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right), \left(\frac{-\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ etc., is bijective and its range is R.

Thus $\tan^{-1}x$ can be defined as a function whose domain is R and range could be any of the intervals $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right), \left(\frac{-\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and soon.

Assertion (A): Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$

Reason (R): Principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation:

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$$

$$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$$

$$\cot y = \frac{-1}{\sqrt{3}} = -\cot\left(\frac{\pi}{3}\right) = \cot\left(\pi - \frac{\pi}{3}\right) = \cot\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \quad \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{2\pi}{3}$$

Hence Assertion is correct and Reason is incorrect.

Assertion (A): Range of $\cot^{-1}x$ is $(0, \pi)$

Reason (R): Domain of $\tan^{-1}x$ is R.

Ans. Option (B) is correct.

Assertion (A): Principal value of $\cos^{-1}(1)$ is π

Reason (R): Value of $\cos 0°$ is 1

Ans. Option (D) is correct.

Explanation: In case of Assertion:

$$\cos^{-1}(1) = y$$

$$\cos y = 1$$

$$\cos y = \cos 0° \qquad [\because \cos 0° = 1]$$

$$\therefore \quad y = 0$$

$\Rightarrow$ Principal value of $\cos^{-1}(1)$ is 0

Hence Assertion is in correct.

Reason is correct.

Assertion (A) : Domain of $f(x) = \sin^{-1} x + \cos x$ is $[-1, 1]$.

Reason (R) : Domain of a function is the set of all possible values for which function will be defined.

Assertion (A) : Function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sin x$ is not a bijection.

Reason (R) : A function $f : A \to B$ is said to be bijection if it is one-one and onto.

Assertion (A) : Principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

Reason (R) : $\tan^{-1} : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so for any $x \in \mathbb{R}$, $\tan^{-1}(x)$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Assertion (A) : $\sin^{-1}(-x) = -\sin^{-1} x;\ x \in [-1, 1]$

Reason (R) : $\sin^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is a bijection map.

Answers